AERODYNAMICS AND HEAT EXCHANGE OF A SWIRLED STREAM IN A CYLINDRICAL CHANNEL

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The results of an investigation and the method of calculation of aerodynamics and heat transfer in short cylindrical chambers with swirled heat-transfer agents are presented.

Swirling of the heat-transfer agent is widely used to intensify processes of heat and mass exchange in various devices. The variety of technological purposes of these devices determines the special features of their geometry, methods of swirling of the heat-transfer agent, and systems of input and output of the gases. In the paper we consider the special features of gas motion and heat transfer in a short cylindrical chamber with a stream-swirling generator in the form of tangential channels. The inlet channels were located near one end of the chamber. Output of the gases was through windows on its lateral surface at the opposite end (Fig. 1). Chambers of this construction are widely used in engineering, as heating and cooling devices [1], among others, but there are practically no recommendations on their calculation.

The experimental part of the work was carried out on a stand whose main elements are models of the cylindrical chamber. Their dimensionless length $\bar{L}$ was varied from 0.69 to 2.63 in the tests. The inside diameters were 201 and 310 mm. The bodies of the models were assembled from individual sections tightly connected to each other. Air was supplied to the chambers from one or from two diametrically opposite directions. The number of peripheral exit windows was varied from one to eight. The relative area $\bar{f}_{in}$ of the inlet slots was varied from $1.42 \cdot 10^{-2}$ to $9.54 \cdot 10^{-2}$; the height $\bar{h}_{in}$ of the inlet slots was varied from $3.23 \cdot 10^{-2}$ to $12.90 \cdot 10^{-2}$; the output area (of the outlet windows) $\bar{f}_{out}$ was varied from $2.39 \cdot 10^{-2}$ to $9.54 \cdot 10^{-2}$.

The velocity and pressure distributions in the main part of the chamber volume were measured with three-channel cylindrical and five-channel spherical probes by the usual method without causing significant disturbances in the stream. The velocity distributions in the wall boundary layer were taken with a special plane three-channel microtube. Outside dimensions of the receiver opening of the central channel of the microtube: height 0.39 mm, width 1.3 mm; inside dimensions: height 0.2 mm, width 1.23 mm. The profile of the tip of the microtube was bent on a template so as to assure that its receiver part lay flush against the side surface of the chamber. The reading of the transverse coordinate from the wall was made from the instant contact was broken in the electric circuit between the tip of the tube and the surface of the chamber.

The experimental investigation of the stress of surface friction on the side surface of the chamber was made by Preston's method [2], while heat transfer was investigated, as in [3], by the method of variation of the aggregate state of the heating agent — the condensation of slightly superheated steam. A thin-walled steam calorimeter was mounted on one of the sections of the body of the model. The longitudinal size of the working section of the calorimeter was 42 mm and the transverse size (along the perimeter) was 103 mm. To eliminate heat leaks along the chamber wall by heat conduction, the working section of the calorimeter was placed at the center of the steam jacket, while Textolite gaskets were mounted between the sections of the model with the calorimeter and the other sections. The temperature of the heat-transfer surface was monitored with copper—constantan thermocouples calked to it.

These investigations allowed us to establish that, by somewhat simplifying the scheme of gas flow, the swirled streams in the chambers under consideration can be represented in the form of a peculiar plane semiconfined jet on a curved surface. The jet propagates under the

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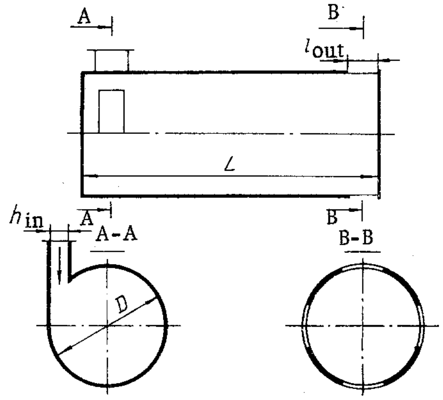

Fig. 1. Diagram of the cylindrical chamber.

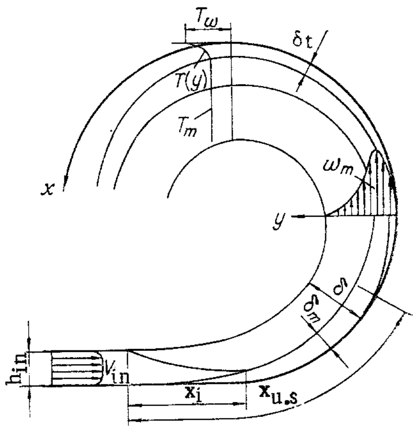

Fig. 2. Diagram of jet propagation near the wall.

influence of longitudinal and transverse pressure gradients. Upon entering the chamber the jet, making one revolution in the inlet plane, departs from it and then propagates toward the outlet windows with a constant swirling angle γ. The asymmetry of the stream along the perimeter, very pronounced in the inlet cross section, disappears practically at once below it in the main section of flow, for which γ = const.

For the flow analysis we direct the x coordinate along the surface of the chamber along the curved trajectory of motion of the jet and the y axis along the normal to it (Fig. 2). We write the system of equations of the plane turbulent boundary layer for an incompressible gas on a curved surface with a constant radius of longitudinal curvature R in the form [4]

$$\rho w \frac{\partial w}{\partial x} + \rho v \frac{\partial}{\partial y}\left[\left(1-\frac{y}{R}\right) w\right] = -\frac{\partial P}{\partial x} + \left(1-\frac{y}{R}\right)\frac{\partial \tau}{\partial y} - \frac{2\tau}{R};$$

$$-\frac{\rho w^2}{R\left(1-\frac{y}{R}\right)} = \frac{\partial P}{\partial y}; \tag{1}$$

$$\frac{\partial w}{\partial x} + \frac{\partial}{\partial y}\left[\left(1-\frac{y}{R}\right) v\right] = 0.$$

The boundary conditions of the problem are

$$y = 0, \quad w = v = 0, \quad \tau = \tau_w;$$

$$y = \delta_m, \quad w = w_m, \quad \tau = 0;$$
$$y = \delta, \quad w = 0, \quad \tau = 0.$$

The radius of longitudinal curvature R of the surface can be calculated from the well-known formula for the curvature of a helical line.

The central part of the chamber is occupied by a mass of gases of relatively low activity from the dynamic point of view, so that near the chamber axis we can use the condition $\partial P/\partial x = 0$. We transform the first equation of the system (1) using the second and third equations, and then we integrate over y from zero to δ_m and δ. As a result of the integration and an estimate of the order of the terms [5], we obtain

$$\frac{d}{dx}\int_0^{\delta_m} w^2 dy - w_m\left(1-\frac{\delta_m}{R}\right)\frac{d}{dx}\int_0^{\delta_m} w dy - \frac{\delta_m}{R}\frac{d}{dx}\int_{\delta}^{\delta_m} w^2 dy - \frac{\delta_m}{R} w_m^2 \frac{d\delta_m}{dx} = -\frac{\tau_w}{\rho}; \tag{2}$$

$$\frac{d}{dx}\int_0^{\delta_m} w^2 dy + \frac{d}{dx}\int_{\delta_m}^{\delta} w^2 dy + \frac{\tau_w}{\rho} = 0. \tag{3}$$

To close the system (2), (3) we use the connection [6]

$$\delta_m/y_{0.5m} = 0.15, \tag{4}$$

well confirmed by our tests, as well as the dependence obtained;

$$c_f = \frac{2\tau_w}{\rho w_m^2} = 0.0183\,(w_m\delta_m/\nu)^{-2/15}. \tag{5}$$

In the region of the wall boundary layer the experimental distributions of the longitudinal component of the stream velocity could be generalized well by the usual power-law relation with an exponent of 1/14, while in the jet part they could be generalized by the well-known Abramovich–Schlichting equation [7].

The expressions (2) and (3), with allowance for (4) and (5) and for the equations describing the velocity distributions in both parts of the jet stream, after calculation of the integrals and certain transformations, were replaced by a system of two equations, which were then solved numerically. As a result, we obtained calculating equations for determining the thickness of the wall boundary layer and the velocity at its boundary:

$$\bar{\delta}_m = \delta_m/h_{\text{in}} = 0.016\,\text{Re}_m^{-0.113}\bar{x}^{0.943}\bar{R}^{0.058}, \tag{6}$$

$$\bar{w}_m = \frac{w_m}{w_{m_\text{i}}} = \frac{B}{\bar{x}^{0.53}}, \tag{7}$$

where $\bar{x} = x/h_{\text{in}}$. (Since flow at the inner concave surface of the cylindrical chamber is being investigated in the paper, in Eq. (6) and below we consider only the absolute value of the radius of curvature.)

The numerical value of B in Eq. (7) is determined from the condition $\bar{w}_m = 1$ at $\bar{x} = \bar{x}_\text{i}$. As is known, the initial conditions of discharge of a jet have considerable influence on its dynamic characteristics. The agreement between experimental data obtained in the work and data calculated from Eq. (7) improves if we introduce into the analysis a coefficient β of correction for momentum, allowing for the nonuniformity of the velocity profile in the inlet channel, as well as the polar distance x_0, connected with the finite size of the source of the jet. Then Eq. (7) takes the form

$$\bar{w} = \frac{B\beta}{\bar{x}_1^{0.53}}, \tag{8}$$

where $\bar{x}_1 = \bar{x} - \bar{x}_0$.

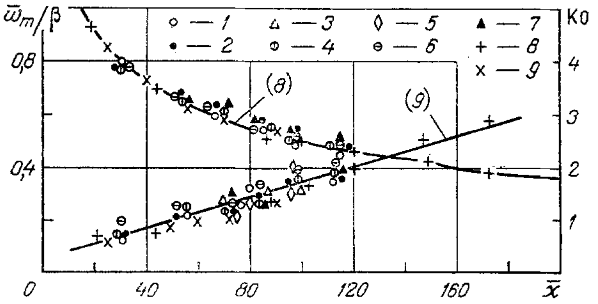

Fig. 3. Comparison of test data with the calculated relations (8) and (9), Ko = $\bar{\delta}_m/\mathrm{Re}^{-0.10}\bar{R}^{0.05}(B\beta)^{-0.10}$. Parameters of the cylindrical chamber 1) $\bar{f}_{in} = 1.42\cdot10^{-2}$; 2) $2.84\cdot10^{-2}$; 3) $4.26\cdot10^{-2}$ for $\bar{h}_{in} = 7.46\cdot10^{-2}$ and $a = 1$; 4) $\bar{f}_{in} = 2.84\cdot10^{-2}$; 5) $4.26\cdot10^{-2}$; 6) $5.68\cdot10^{-2}$; 7) $8.51\cdot10^{-2}$ for $\bar{h}_{in} = 7.46\cdot10^{-2}$ and $a = 2$; 8) $\bar{h}_{in} = 4.98\cdot10^{-2}$; 9) $9.95\cdot10^{-2}$ for $\bar{f}_{in} = 1.42\cdot10^{-2}$ and $a = 1$.

Substituting (8) into (6), we determine the variation of the thickness of the wall boundary layer in the explicit form

$$\bar{\delta}_m = 0.0243\mathrm{Re}^{-0.10}\bar{x}_1^{0.9}\bar{R}^{0.05}(B\beta)^{-0.10}. \tag{9}$$

In Fig. 3 the calculated relations (8) and (9) are compared with the test data. From the test conditions $\bar{x}_0 = -15$, $\bar{x}_i = 15$; $B = 6.07$; $\beta = 1.9\bar{f}_{in}^{0.15}$; $w_{mi} = 0.92\bar{f}_{in}^{0.35}\bar{h}_{in}^{0.5}V_{in}$; $\gamma = 18°$; $\bar{R}$ = 3.88-15.5. The tests were made with Gö = 10.8-91.6 and Re = $(0.87\text{-}6.7)\cdot10^4$, i.e., in a developed turbulent stream, when the influence of Taylor–Görtler vortices on the velocity distribution in the wall boundary layer and its thickness is insignificant. Determining the $\bar{x}$ coordinate through the swirling angle γ, we can calculate the velocity characteristic and the geometrical characteristic of the stream at any point of the chamber using Eqs. (8) and (9), the velocity distributions, and the connection (4).

We calculate the heat transfer using a modified Reynolds analogy [8]

$$\mathrm{St} = 0.5c_f\,\mathrm{Pr}_{\mathrm{tb}}^{-1}(\delta_t/\delta_m)^{-n}. \tag{10}$$

We determine the ratio of the thicknesses of the thermal and hydrodynamic boundary layers introduced in Eq. (10) using the integral energy equation

$$\frac{d}{dx}\int_0^{\delta_t} w(T_w - T)\,dy = \frac{q_w}{\rho c_p}, \tag{11}$$

Eqs. (8) and (9), and the assumption that the distribution of excess temperature in the wall thermal boundary layer is similar to the velocity distribution in the hydrodynamic boundary layer and is also described by a power-law distribution with an exponent of 1/14:

$$\delta_t/\delta_m = 1.586\mathrm{Re}^{0.018}\,\mathrm{Pr}_{\mathrm{tb}}^{-7/8}\bar{x}_1^{0.044}\bar{R}^{-0.05}[1-(x_{\mathrm{u.s}}/x_1)^{0.445}]^{7/8}(B\beta)^{0.018}. \tag{12}$$

Substituting Eq. (12) into (10), we obtain an equation for calculating the local heat-transfer coefficients along the trajectory of motion of the jet stream in the main section:

$$\mathrm{Nu} = 0.0147\,\mathrm{Re}^{0.88}\,\mathrm{Pr}\,\mathrm{Pr}_{\mathrm{tb}}^{-15/16}\bar{x}_1^{-0.58}\bar{R}^{-0.003}[1-(x_{\mathrm{u.s}}/x_1)^{0.445}]^{-1/16}(B\beta)^{0.88}. \tag{13}$$

In Fig. 4 the calculated relation (13) is compared with test data (Pr = 0.7) for different lengths of the initial unheated section. In the comparison the turbulent Prandtl number was taken as 0.75, as is done for jets on a plane surface [7]. The test data were obtained for $x_{\mathrm{u.s}}/h_{in}$ = 57-186 and $\mathrm{Re}\cdot10^{-4}$ = 0.87-6.7. The agreement of the calculated and experimental results can be acknowledged as quite satisfactory. However, it must be noted that for

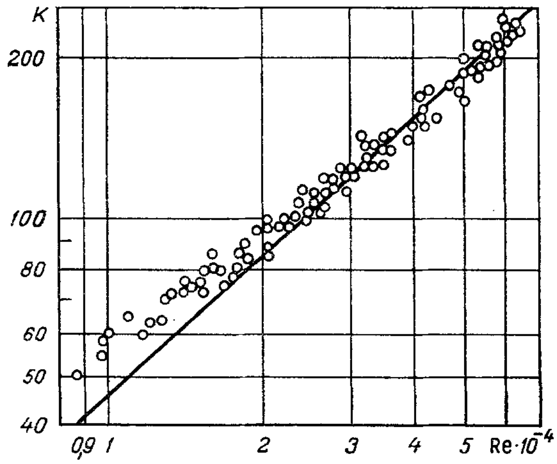

Fig. 4. Comparison of the calculated relation (13) with test data for different lengths of the initial unheated section, $K = \mathrm{Nu}/\bar{x}_1^{-0.58}\bar{R}^{-0.003}\left[1 - \left(\frac{x_{\mathrm{u.s}}}{x_1}\right)^{0.445}\right]^{-1/16}(B\beta)^{0.88}$.

Re < $3\cdot 10^4$ the test data lie somewhat higher than the relation (13). With a scatter of ±12.6% for a confidence coefficient of 0.95 they are well generalized by the equation

$$\mathrm{Nu} = 0.096\,\mathrm{Re}^{0.7}\,\mathrm{Pr}\,\mathrm{Pr}_{\mathrm{tb}}^{-15/16}\bar{x}_1^{-0.58}\bar{R}^{-0.003}\left[1 - (x_{\mathrm{u.s}}/x_1)^{0.445}\right]^{-1/16}(B\beta)^{0.88}. \tag{14}$$

In the absence of an unheated section ($x_{\mathrm{u.s}} = 0$) Eqs. (13) and (14) change into the calculating equations for the case when the dynamic and thermal boundary layers start to form simultaneously.

To estimate the reliability of the results obtained, heat transfer on the side surface of the chamber was also calculated by a different method, using universal test distributions of the velocity in the wall boundary layer. In the region of the viscous sublayer ($0 \leqslant y^+ \leqslant 5$) and in the buffer zone ($5 \leqslant y^+ \leqslant 30$) the distributions of velocity correspond to its variation in wall jets on a plane surface [7]: for $0 \leqslant y^+ \leqslant 5$

$$w^+ = y^+; \tag{15}$$

for $5 \leqslant y^+ \leqslant 30$

$$w^+ = 6.24\ln y^+ - 5. \tag{16}$$

In the turbulent core (for $30 \leqslant y^+ \leqslant \delta_m^+$, where δ_m^+ is the conventional thickness of the wall boundary layer, $\delta_m^+ \approx 400$)

$$w^+ = 1.23\ln y^+ + 12.04. \tag{17}$$

A calculation of the temperature profile [9] using the velocity distributions (15)-(17) allows us to determine the temperature head in the wall boundary layer,

$$\vartheta_m^+ = \frac{(T_w - T_m)\rho c_p}{q_w}\sqrt{\frac{\tau_w}{\rho}} = 6.24\,\mathrm{Pr}_{\mathrm{tb}}\left(\ln\frac{1 + 3.81\,\mathrm{Pr}/\mathrm{Pr}_{\mathrm{tb}}}{1 - 0.2\,\mathrm{Pr}/\mathrm{Pr}_{\mathrm{tb}}} + 0.2\ln\frac{\delta_m^+}{30}\right) + 5\,\mathrm{Pr}. \tag{18}$$

The known values of the heat flux at the wall and the temperature head make it possible to obtain a calculating heat-transfer equation,

$$\mathrm{Nu} = \frac{\mathrm{Pr}}{\vartheta_m^+} \sqrt{\frac{c_f}{2}}\, \mathrm{Re}_m \bar{\delta}_m^{-1}. \tag{19}$$

Substituting (5), (8), (9), and (18) into Eq. (19), we convert it to the form

$$\mathrm{Nu} = \frac{0,122\mathrm{Re}^{0,94}\,\mathrm{Pr}\bar{x}_\mathrm{I}^{-0,56}\bar{R}^{-0,003}\,(B\beta)^{0,94}}{6,24\,\mathrm{Pr_{tb}}\left(\ln\dfrac{1+3,81\,\mathrm{Pr/Pr_{tb}}}{1-0,2\,\mathrm{Pr/Pr_{tb}}}+0,518\right)+5\,\mathrm{Pr}}. \tag{20}$$

A comparison of the relations (13) for $x_{u.s} = 0$ and (20) in the investigated range of Reynolds numbers shows that the maximum discrepancy between them does not exceed 11%.

NOTATION

D, diameter of the cylindrical chamber; $\bar{L} = L/D$, dimensionless length of the working volume of the cylindrical chamber; $\bar{h}_{in} = h_{in}/D$, dimensionless height of the inlet channel; $\bar{f}_{in} = 4f_{in}/\pi D^2$, $\bar{f}_{out} = 4f_{out}/\pi D^2$, dimensionless areas of the stream inlet and outlet, respecitvely; x, y, longitudinal and transverse coordinates; γ, stream swirling angle; $\bar{R} = R/h_{in}$, dimensionless radius of curvature; w, v, velocity components in the directions of the x and y axes, respectively; w_m, maximum velocity in the direction of the x axis; $y_{0.5m}$, value of y where the velocity w equals half its maximum value; x_i, length of the initial section of the semiconfined jet; x_0, polar distance of the jet; w_{mi}, maximum velocity at the end of the initial section of the semiconfined jet; V_{in}, average stream velocity in the inlet channels; P, pressure; ρ, stream density; τ, shear stress; τ_w, wall shear stress; c_f, frictional resistance coefficient; δ, thickness of the jet; δ_m, thickness of the wall hydrodynamic boundary layer; δ^{xx}, momentum thickness; δ_t, thickness of wall thermal boundary layer; β, correction coefficient for momentum; T, temperature; T_w, temperature of chamber surface; c_p, isobaric heat capacity; q_w, heat-flux density at the surface; ν, kinematic viscosity coefficient; a, coefficient of thermal diffusivity; λ, coefficient of thermal conductivity; α, local heat-transfer coefficient; ε_σ, ε_q, kinematic coefficients of turbulent momentum and heat transfer, respectively; $y^+ = \frac{y}{\nu}\sqrt{\frac{\tau_w}{\rho}}$, dimensionless coordinate; $w^+ = w/\sqrt{\tau_w/\rho}$, dimensionless velocity; $\vartheta_m^+ = (T_w - T_m)\rho c_p \sqrt{\tau_w/\rho}/q_w$, dimensionless temperature head in the wall boundary layer; $\mathrm{Re} = w_{mi}h_{in}/\nu$, $\mathrm{Re}_m = w_m\delta_m/\nu$, Reynolds numbers; $\mathrm{Pr} = \nu/a$, Prandtl number; $\mathrm{Pr_{tb}} = \varepsilon_\sigma/\varepsilon_q$, turbulent Prandtl number; $\mathrm{Nu} = \alpha h_{in}/\lambda$, Nusselt number; $\mathrm{St} = \alpha/\rho c_p w_m$, Stanton number; $\mathrm{Gö} = (w_m\delta^{xx}/\nu)\sqrt{\delta^{xx}/R}$, Görtler number.

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NONLINEAR WAVES ON THE SURFACE OF A FREELY FLOWING VERTICAL LIQUID FILM

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The nonlinear equation describing nonstationary waves on the surface of a freely flowing vertical liquid film has been investigated by a perturbation theory method.

1. The wave flow regimes of thin films on a vertical wall were investigated both experimentally [1-4] and theoretically [4-9] in many studies. Experiment shows that laminar flow of a liquid film is unstable, starting with very small Reynolds numbers. The instability leads to generation of periodic waves on the surface of the film, whose amplitude increases with propagation, and quickly departs from the stationary value. To determine the characteristics of stationary waves, various assumptions on the wave flow regime are usually used in theoretical studies. Thus, in the first problem investigated on wave flow of a vertical liquid film, Kapitza [6] assumed minimal viscous energy dissipation for the wave realized. An assumption was introduced [7] on "optimality" of the wave regime, i.e., minimality (for a given liquid discharge) of the mean film thickness. It was assumed in [4] that only "maximum growth waves" are realized experimentally, for which the amplitude increment is maximum. A problem was subsequently solved [8], where it was taken into account that in the stationary regime the amplitude increment corresponding to the stationary value of the wave number vanishes. At the same time, for all other wave number values the increment must be negative. The use of various assumptions of this nature, such as in [4, 6, 7], often leads to good agreement with experiment, but is, in our opinion, somewhat artificial. It seems to us that the wave characteristics of established flows must be obtained naturally from the solution of the nonstationary nonlinear equation describing the wave formation. In the present paper an attempt was made to solve this problem, using the method of slowly varying parameters, developed in detail by Bogolyubov and Mitropol'skii [10] for nonlinear system oscillations. This method was generalized in [11-16] so as to investigate nonlinear wave processes.

In the region of large Reynols numbers, when $\mathrm{Re}(h_0/\lambda) \gg 1$, the original equation for the film thickness h(x, t) (Fig. 1) is

$$\frac{\partial h}{\partial t} + 1.7v_0\frac{\partial h}{\partial x} + 2.3v_0\frac{h-h_0}{h_0}\frac{\partial h}{\partial x} - \frac{\sigma h_0}{v_0 d}\frac{\partial^3 h}{\partial x^3} = \frac{3\nu}{h_0^2 v_0}\int\left(\frac{\partial h}{\partial t} + 3v_0\frac{\partial h}{\partial x}\right)dx, \quad (1)$$

where $h_0 = \sqrt[3]{\frac{3\nu}{g}Q}$; $v_0 = \frac{Q}{h_0}$.

The equation given was obtained in [4]. Its linearized variant is also contained in the monograph [9]. For further study of this equation (see [4]), in the right-hand side we replaced the time derivative $\frac{\partial h}{\partial t}$ by $-c\frac{\partial h}{\partial x} \approx -1.7v_0\frac{\partial h}{\partial x}$, and carried out the integration. This

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